Non-Fermi liquids induced by U(1) gauge field interactions: a functional renormalization group analysis

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Outline

1. Introduction
   - Biases and guesswork in modelling of non-Fermi liquids.
   - Possible remedy: symmetry-constraints. The Reizer instability.
2. The renormalization group for NFLs
   - Brief introduction to the renormalization group.
   - Functional RG: how to handle Landau-damping consistently.
3. Symmetries in the functional renormalization group
   - (Modified) Ward-Takahashi identities.
   - Our effective action, and constraints on the RG flow.
4. RG Fixed Point
   - Unconstrained vs constrained flow.
5. Discussion and summary

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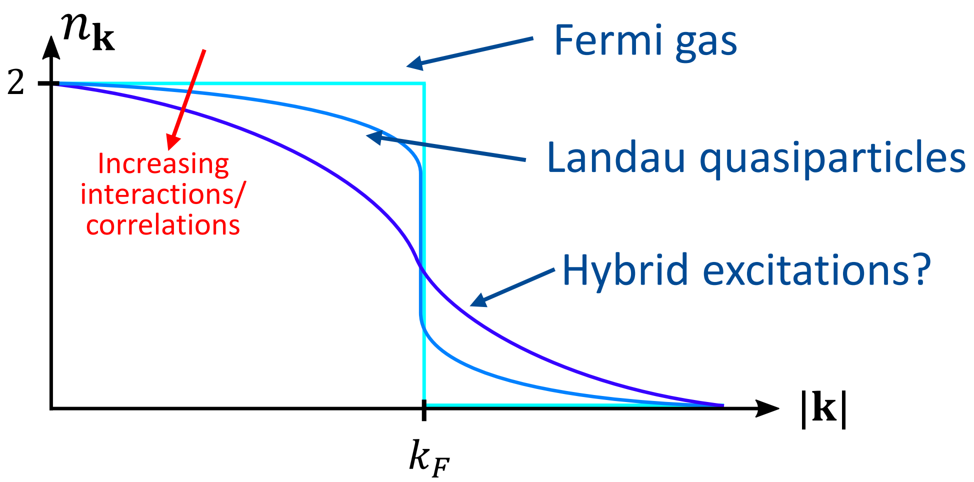
1. Introduction

- Developing consistent theories of non-Fermi liquids (NFLs) in two spatial dimensions remains a key challenge to condensed matter physics.
    - Hertz-Millis theory is useful for some universality classes in 2-D, but fails in many important cases [1,2].
    - Various other approaches exist (Yukawa-SYK [3], QMC [4], etc) but only give insight into certain classes of problems.

- Long-term goal: systematic understanding of the low-energy theory of these important cases.

[1] Abanov & Chubukov, PRL **93**, 255702 (2004) [2] Thier & Metzner, PRB **84**, 155133 (2011)
[3] Patel, Guo, Esterlis & Sachdev, Science **381**, 790 (2023) [4] Xu, Sun, Schattner, Berg & Meng, PRX **7**, 031058 (2017)

- Why is it so hard?
  - Breakdown of quasiparticle picture → little known about form of low-energy effective theory, or what the fermions do at criticality.



  - Strong interactions/correlations in 2-D → failure of “usual” methods (Hubbard-Stratonovich transformation [1,2], $1/N$-expansion [5]).

[1] Abanov & Chubukov, PRL **93**, 255702 (2004) [2] Thier & Metzner, PRB **84**, 155133 (2011)
[5] Lee, PRB **80**, 165102 (2009)

- As a result, modelling procedures often include biases and uncontrolled approximations (e.g. Hertz-Millis theory assumes Fermi-liquid form for fermion propagator [6], holography assumes AdS/CFT correspondence and postulates effective field theory [7], etc).

  → loss of predictive power.

- Partial remedy: constrain modelling using exact (nonperturbative) identities that survive renormalization, e.g. constraints from ‘t Hooft anomalies and (gauge) symmetries.

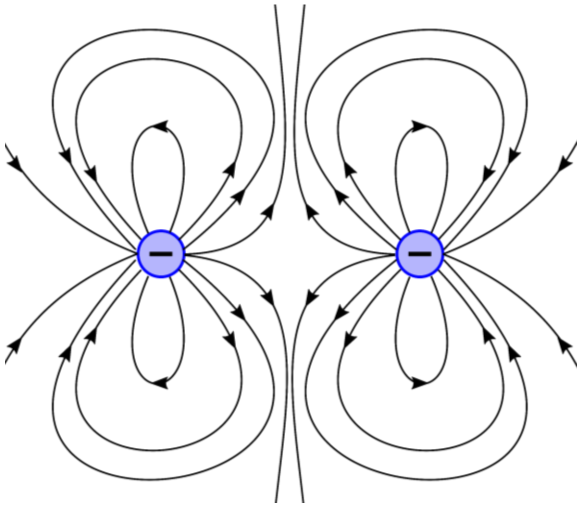
(also c.f. Zhengyan Shi’s talk on Tuesday)

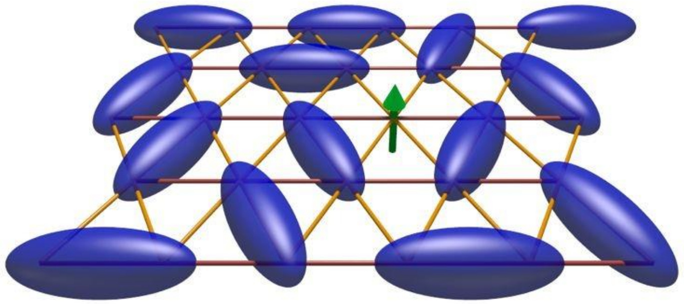
[6] Hertz, PRB **14**, 1165 (1976) [7] Sachdev, J. Stat. Mech. (2010) P11022

- Gauge symmetries are implemented through exact relations between correlation functions, called Ward-Takahashi identities

  $\rightarrow$ good for constraining modelling-procedures/Ansatzes.

- Simplest example: $\boxed{\mathrm{U}(1) \text{ gauge field interacting with circular Fermi surface.}}$ Could be electromagnetic field, or emergent gauge field, as in spin-liquids.





M. Vojta, Racah Institute of Physics colloquium, 16/11/2020

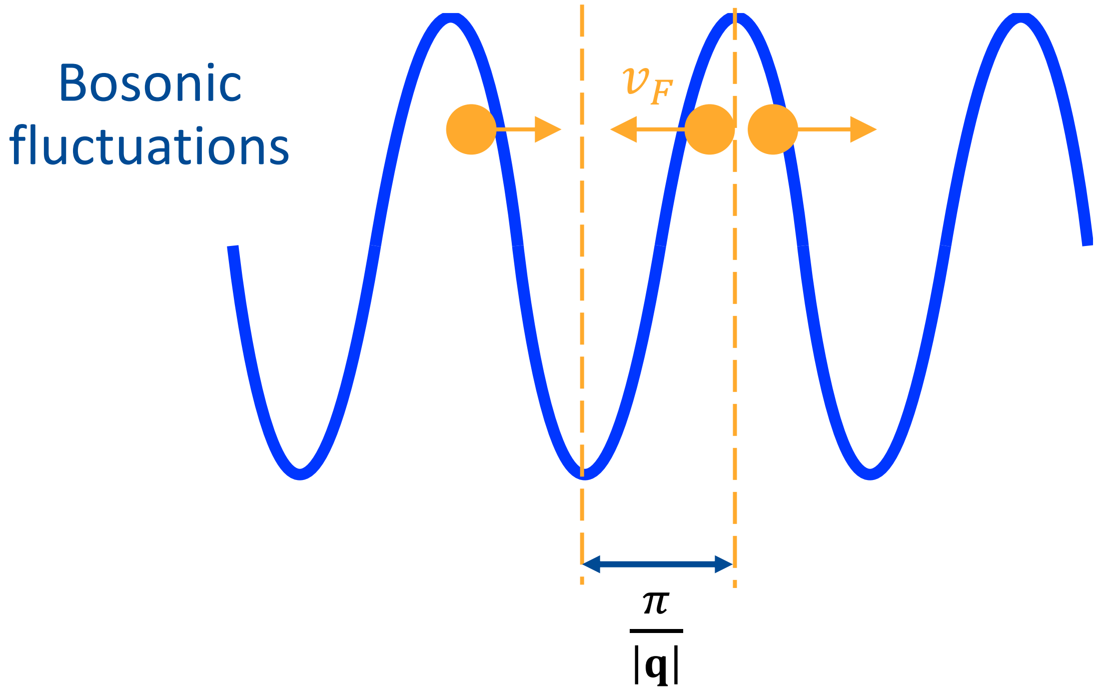
- Well-known that the magnetic vector potential $\mathbf{A}$ is unscreened by the particle-hole continuum → long-range, singular interactions between electrons → destabilizes the Fermi liquid at (very) low energies ("Reizer instability" [8]).

- Previous approaches have largely been perturbative, and not paid close attention to symmetry-constraints [9,10]

  → use functional renormalization group augmented by Ward-Takahashi identities.

[8] Reizer, PRB **40**, 11571 (1989) [9] Holder & Metzner, PRB **92**, 041112(R) (2015)
[10] Mandal, PRR **2**, 043277 (2020)

- Main obstacle/ challenge: reproduce Landau-damping while preserving gauge symmetry.



Decay-time $\tau \sim \frac{\pi}{v_F|\mathbf{q}|}$

$\rightarrow$ Inverse propagator changes from

$$\Omega^2 + c^2|\mathbf{q}|^2 \rightarrow \Omega^2 + c^2|\mathbf{q}|^2 + C\,\frac{|\Omega|}{v_F|\mathbf{q}|},$$

so $\Omega \sim |\mathbf{q}|^3$,

or more generally

$$\Omega \sim |\mathbf{q}|^{z_b}, z_b > 1$$

- Turns out to be difficult!

Project goals

1. Non-perturbatively study the Reizer instability, correctly taking account of gauge symmetry.

2. Include Landau-damping in a consistent way.

3. Assess the extent to which enforcing Ward-Takahashi identities changes the properties of the low-energy theory.

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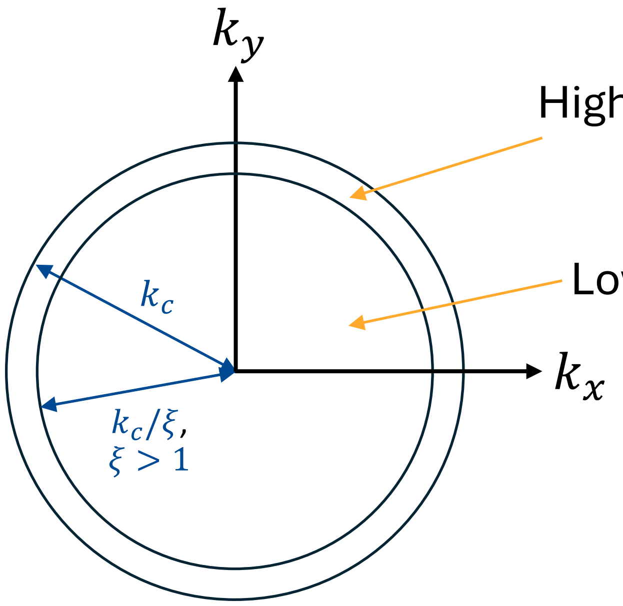
2. The renormalization group for NFLs

- The renormalization group (RG), as applied to condensed matter, is a tool for finding the effective, low-energy theory of a given lattice-model.
- There are many implementations. For translationally-invariant systems: momentum-shell (Wilsonian) RG. Consider bosonic field $\phi(\mathbf{k})$, with UV cutoff k_c:



Partition function $\mathcal{Z} = \int \mathcal{D}\phi_< \mathcal{D}\phi_> \; e^{-S[\phi_<, \phi_>]} \equiv \int \mathcal{D}\phi_< \, e^{-S'[\phi_<]}$

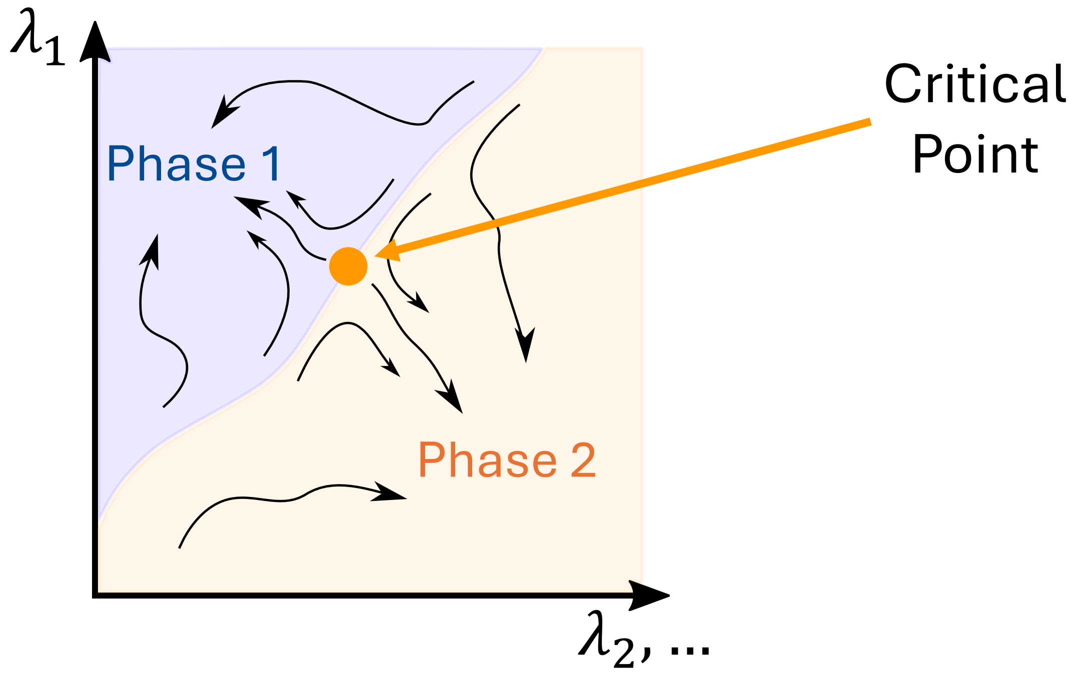
$\rightarrow$ effective action S' for $\phi_<$

- Set $\xi = 1 + dt \rightarrow$ differential change in coupling constants, $d\lambda_i \rightarrow$ RG “flow equations”:

$$\frac{d\lambda_i}{dt} = F_i(\lambda_1, \lambda_2, \dots)$$

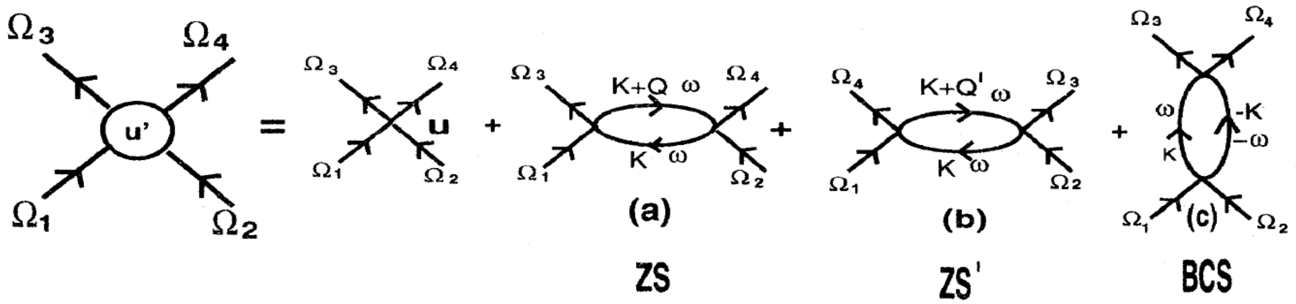
- Flows in λ_i-space generate phase diagrams.

  Fixed points $\left(\frac{d\lambda_i}{dt} = 0\right)$ are either stable (phase) or unstable (critical point).



- Problems with “normal” RG as applied to NFLs:

  a) Perturbative: conventional schemes (like “Shankar RG” [11]) compute β-functions up to a certain loop-order, i.e. rely on small couplings.



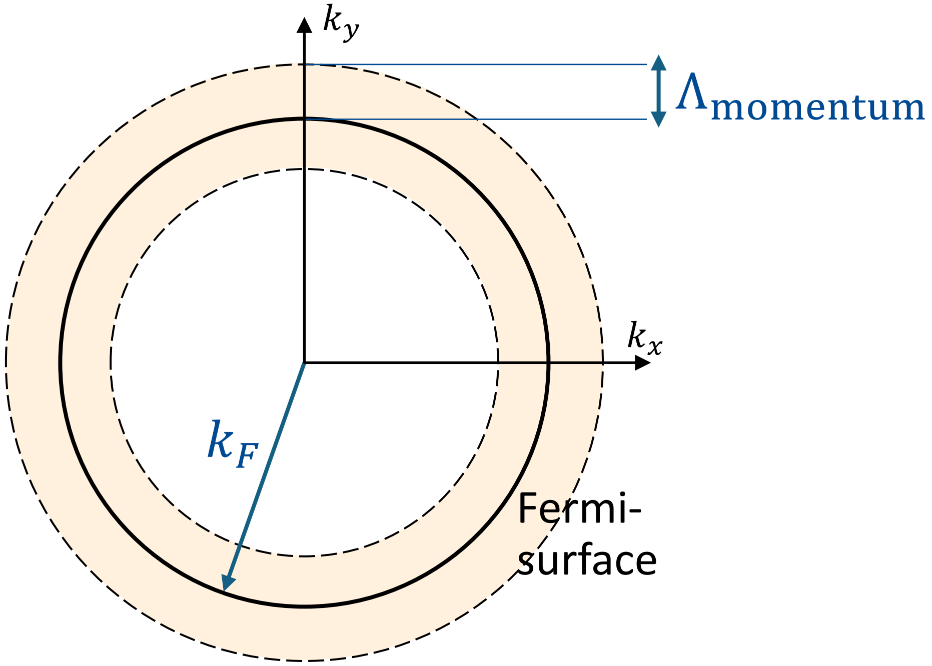
  → bad for NFLs occurring at $\mathcal{O}(1)$ couplings.

[11] Shankar, Rev. Mod. Phys. **66**, 129 (1994)

b) Incompatible momentum-scalings:

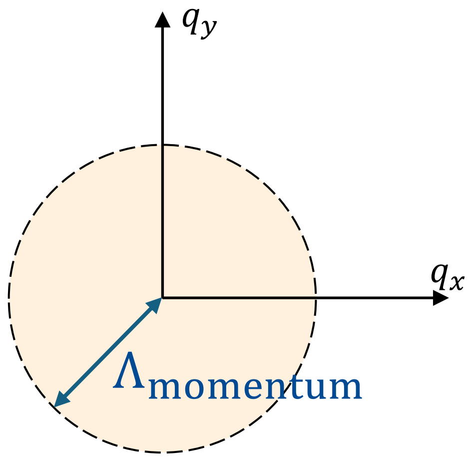
Low-energy fermions live at $|\mathbf{k}| = k_F$, bosons at $\mathbf{q} = \mathbf{0}$:

Fermions



Scale towards
$|\mathbf{k}| = k_F$

Bosons

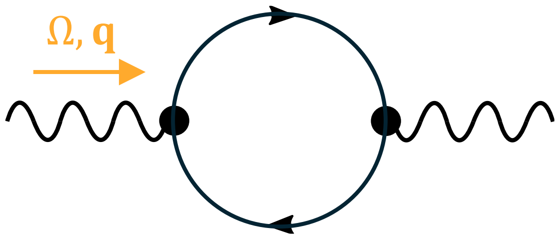


Scale towards
$\mathbf{q} = \mathbf{0}$

c) Flawed description of Landau-damping:

Hertz-Millis-like theories: integrate out the particle-hole bubble to generate Landau-damping:

$$\Delta\left(G_b^{-1}\right)_{1-\mathrm{loop}}(\Omega, \mathbf{q}) =$$



$$= C\,\frac{|\Omega|}{|\mathbf{q}|} + \cdots$$

Bad, as low-energy degrees of freedom are integrated out too early, giving non-local action.

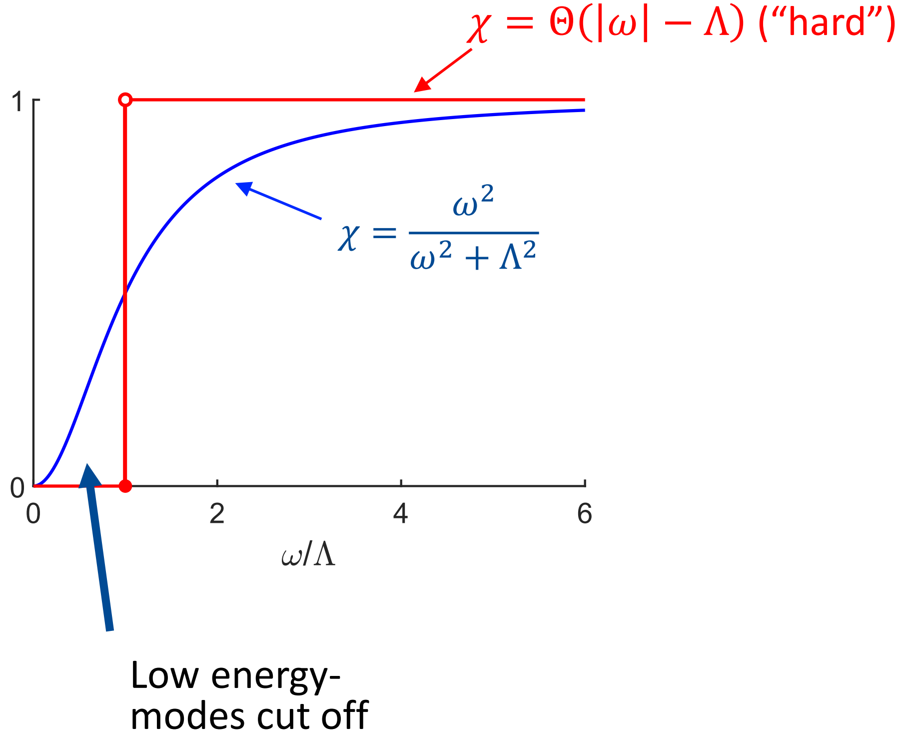
Wilsonian RG with bosons & fermions: action is local at all scales, but no Landau-damping! ($z_b = 1$ throughout [12])

[12] Fitzpatrick, Kachru, Kaplan & Raghu, PRB **88**, 125116 (2013)

Functional Renormalization Group

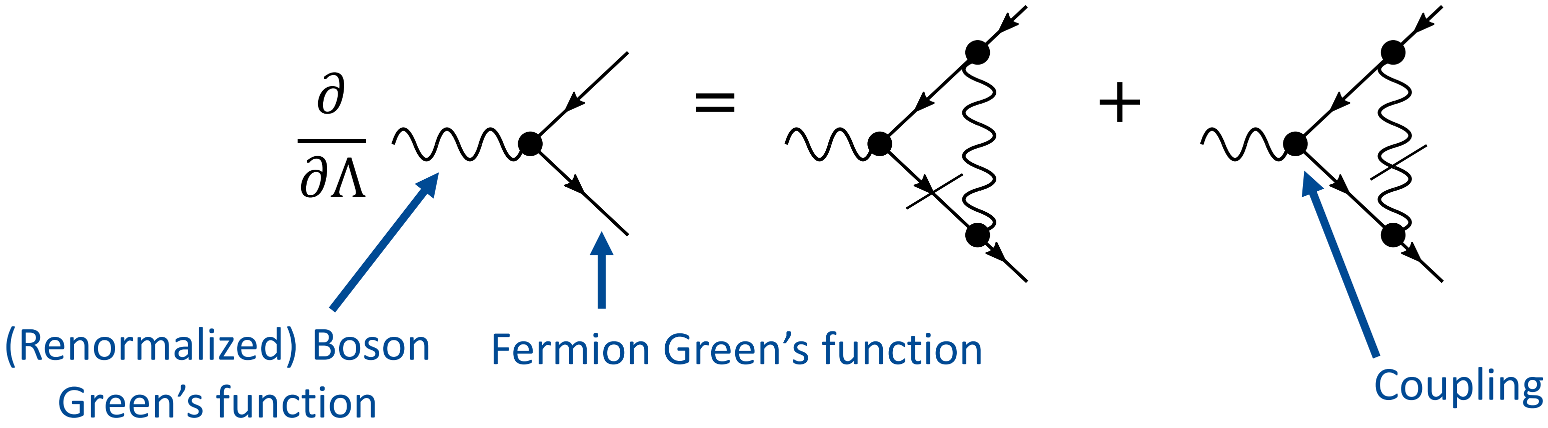
- **Functional RG (fRG)** is well suited to NFLs.
- Here, flow is parametrized by scale Λ, with $\Lambda \to 0$ being the low-energy limit. We use cutoff-functions to regulate IR divergences, e.g. electron propagator:

$$G_f(\omega, \mathbf{k}) \to \chi(\Lambda, \omega, \mathbf{k}) G_f(\omega, \mathbf{k}) \text{, e.g. :}$$



- This gives scale-dependence to the generating functionals

  → flow-equation for effective action → 1-loop-exact flows for correlation functions, e.g.

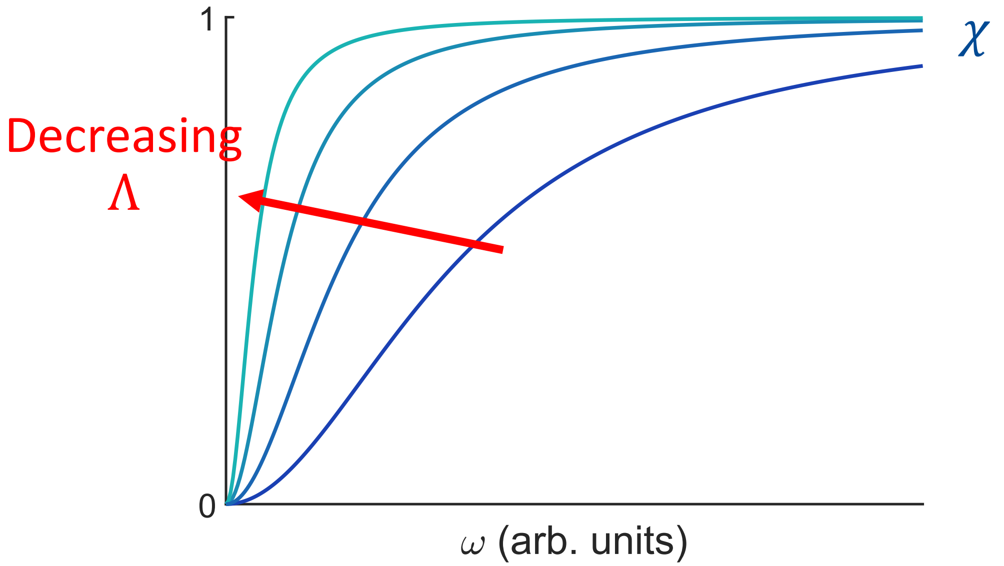


- Our approach [13,14]: $\chi(\Lambda, \omega, \mathbf{k}) = \frac{\omega^2}{\omega^2 + \Lambda^2}$ for fermion propagator (+ some simple cutoff function for bosons). Benefits:

  a) $\boldsymbol{\Lambda}$ **= frequency cutoff**: doesn't suppress small-momentum particle-hole excitations [15]. Also resolves the problem that low-energy fermions & bosons live at different points in momentum space: they both live at zero frequency.

[13] Maier & Strack, PRB **93**, 165114 (2016) [14] Trott & Hooley, PRB **98**, 201113(R) (2018) [15] Honerkamp & Salmhofer, PRB **64**, 184516 (2001)

b) “Soft” cutoff-function: “sees” low-energy degrees of freedom early in flow but suppresses $|\omega| < \Lambda \rightarrow$ Landau-damping develops gradually during flow.



- How do we find $z_{b/f}$? In the scaling-limit, convert frequencies $\omega, \Omega \to \Lambda$, momenta $|\mathbf{k}| - k_F \to \Lambda^{1/z_f}$, $|\mathbf{q}| \to \Lambda^{1/z_b}$.

E.g. fully renormalized fermion propagator:

$$\frac{\omega^2}{\omega^2+\Lambda^2}\,\frac{1}{\underbrace{i\mathcal{A}_\omega\omega} - \underbrace{\mathcal{A}_{\mathbf{k}}(|\mathbf{k}| - k_F)}}$$

At criticality, on-shell:

$$\Lambda\Lambda^{-\eta_\omega} \sim \Lambda^{-\eta_{\mathbf{k}}}\Lambda^{1/z_f} \qquad \left(\eta_{\omega/\mathbf{k}} = -\Lambda\,\partial_\Lambda \log \mathcal{A}_{\omega/\mathbf{k}}\right)$$

$$\Leftrightarrow z_f = \frac{1}{1 - \eta_\omega + \eta_{\mathbf{k}}}$$

Similar for z_b.

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3. Symmetries in the fRG

- In the absence of scale-dependence, (gauge) symmetries are implemented in field theory through Ward-Takahashi identities (WTIs). These are equivalent to invariance of the effective action under gauge-transformation.
- Example: Euclidean action for Fermi-surface coupled to $\mathrm{U}(1)$ gauge field:

$$S = S_{f,0} + S_{b,0} + S_{bf,3} + S_{bf,4}\text{ , with}$$

$$S_{f,0} = \int_k \bar{\psi}(k)\left(-i\omega + v_0(|\mathbf{k}| - k_F)\right)\psi(k)$$

$$S_{b,0} = \frac{1}{2}\int_q A_\mu(-q)\left(q^2 g^{\mu\nu} - q^\mu q^\nu\right)A_\nu(q)$$

$$S_{bf,3} = \int_k \int_q \begin{pmatrix} e \\ \frac{e}{2m}(2\mathbf{k}+\mathbf{q}) \end{pmatrix}^\mu A_\mu(q)\bar{\psi}(k+q)\psi(k)$$

$$S_{bf,4} = \frac{e^2}{2m}\int_k \int_{q_1}\int_{q_2} \mathbf{A}(q_1)\cdot\mathbf{A}(q_2)\bar{\psi}(k+q_1+q_2)\psi(k)$$

$\mu = 0 \leftrightarrow$ Coulomb field ϕ

$\mu = 1{,}2 \leftrightarrow$ Vector potential $\mathbf{A}$

(Coulomb gauge)

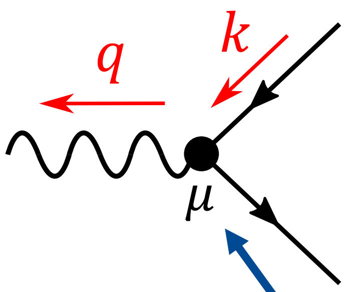
$$\left(\int_k = \int_{\omega,\mathbf{k}}\right)$$

- Up to gauge-fixing, the $\mathrm{U}(1)$ gauge symmetry of this model is expressed, in terms of the effective action Γ, as

$$q_\mu \frac{\delta\Gamma}{\delta A_\mu(-q)} + e\int_k \left[\bar{\psi}(k-q)\frac{\delta\Gamma}{\delta\bar{\psi}(k)} + \frac{\delta\Gamma}{\delta\psi(k)}\psi(k+q)\right] = 0$$

$\rightarrow$ infinite hierarchy of exact relations between <u>renormalized</u> correlation functions, e.g.

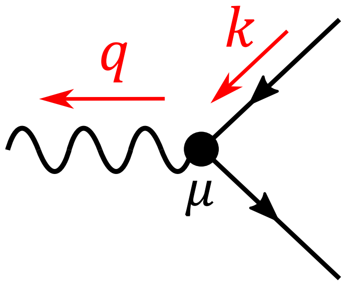
Bare charge

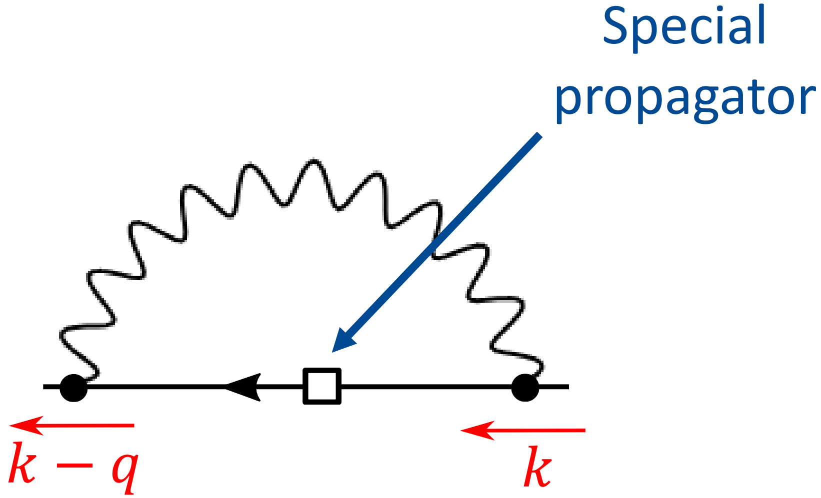
$q^\mu \times$ 

$$= -e\left(G_f^{-1}(k) - G_f^{-1}(k-q)\right)$$

Renormalized

- Complication: cutoff functions break gauge symmetry → **modified Ward-Takahashi identities** [16] (mWTIs), e.g.

$q^\mu \times$ 

$= -e\left(G_f^{-1}(k) - G_f^{-1}(k-q)\right) +$ 

Standard Ward identity

1-loop modification

- Harder to solve due to 1-loop structure, but possible.

[16] H. Gies, in "Renormalization Group and Effective Field Theory Approaches to Many-Body Systems", Chapter 6 (Springer-Verlag Berlin Heidelberg, 2012)

- Our model: our Ansatz for the effective action has

Fermion propagator

$$G_f(\omega, \mathbf{k}) = \frac{\omega^2}{\omega^2 + \Lambda^2} \frac{1}{i\mathcal{A}_\omega \omega - \mathcal{A}_\mathbf{k}(|\mathbf{k}| - k_F)}$$

with quasiparticle weight $\mathcal{Z}_{qp} = 1/\mathcal{A}_\omega$ and Fermi velocity $v = \mathcal{A}_\mathbf{k}/\mathcal{A}_\omega$

A-field propagator

$$G_{\mathbf{A}}^{ij}(\Omega,\mathbf{q}) = \frac{1}{\mathcal{B}_{\mathbf{A}\Omega}(\Omega^2+\Lambda^2)+\mathcal{B}_{\mathbf{A}\mathbf{q}}|\mathbf{q}|^2+M_{\mathbf{A}}^2}\left(\delta^{ij}-\frac{q^i q^j}{|\mathbf{q}|^2}\right)$$

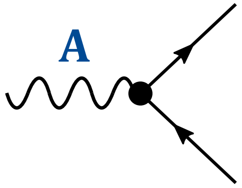
with speed $c^2 = \mathcal{B}_{\mathbf{A}\mathbf{q}}/\mathcal{B}_{\mathbf{A}\Omega}$

Gauge-field masses!

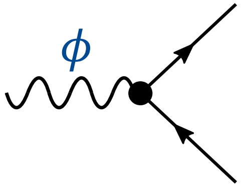
ϕ-field propagator

$$G_{\phi}(\Omega,\mathbf{q}) = -\frac{1}{\mathcal{B}_{\phi\mathbf{q}}(|\mathbf{q}|^2+\Lambda^2/v^2)+M_{\phi}^2}$$

Yukawa vertices

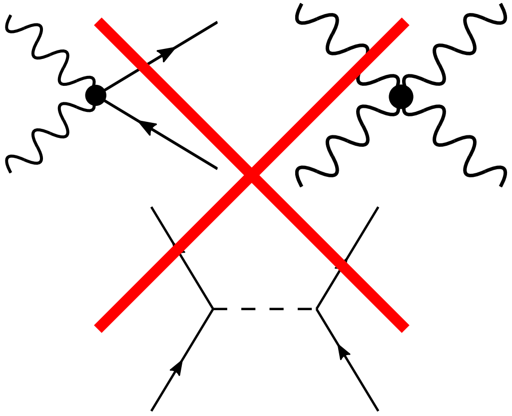


$= g_{\mathbf{A}}$



$= g_{\phi}$

Drop 4-point vertices



- The mWTIs for our model:

Masses:

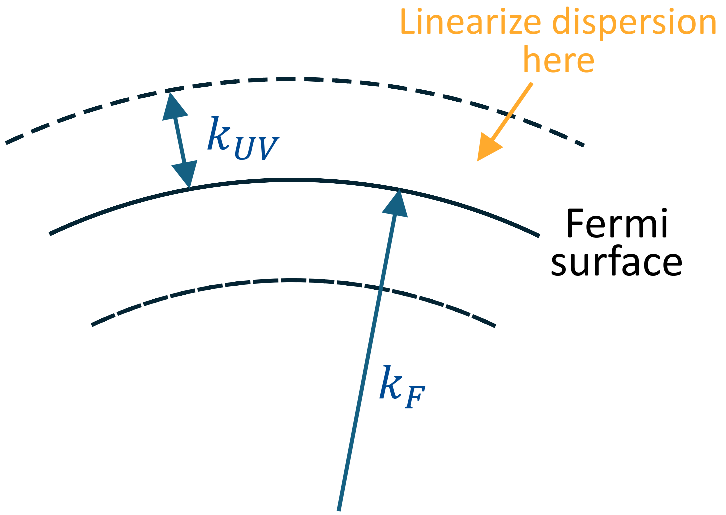
$$M_\phi^2 = \frac{e g_\phi}{\pi \mathcal{A}_{\mathbf{k}}} k_F \qquad M_{\mathbf{A}}^2 = \frac{e g_{\mathbf{A}} N}{4\pi \mathcal{A}_{\mathbf{k}}} \Lambda$$

$$(\sim k_{\mathrm{TF}}^2)$$

Scale set by k_F and e, "large"

Irrelevant → don't need to tune to criticality!

Here, $N = \frac{k_F}{k_{UV}}$, k_{UV} = momentum UV cutoff. N is non-universal → common for theories of NFLs (UV-IR mixing) [17].



[17] Mandal & Lee, PRB **92**, 035141 (2015)

Complicated functions

Yukawa Couplings:

$$g_\phi = \underbrace{e\mathcal{A}_\omega} + g_\phi^2 f_3(\ldots) - g_{\mathbf{A}}^2 f_4(\ldots) \qquad g_{\mathbf{A}} = \underbrace{e\mathcal{A}_{\mathbf{k}}} + g_\phi^2 f_1(\ldots) - g_{\mathbf{A}}^2 f_2(\ldots)$$

Standard QED WTI

Solution-method

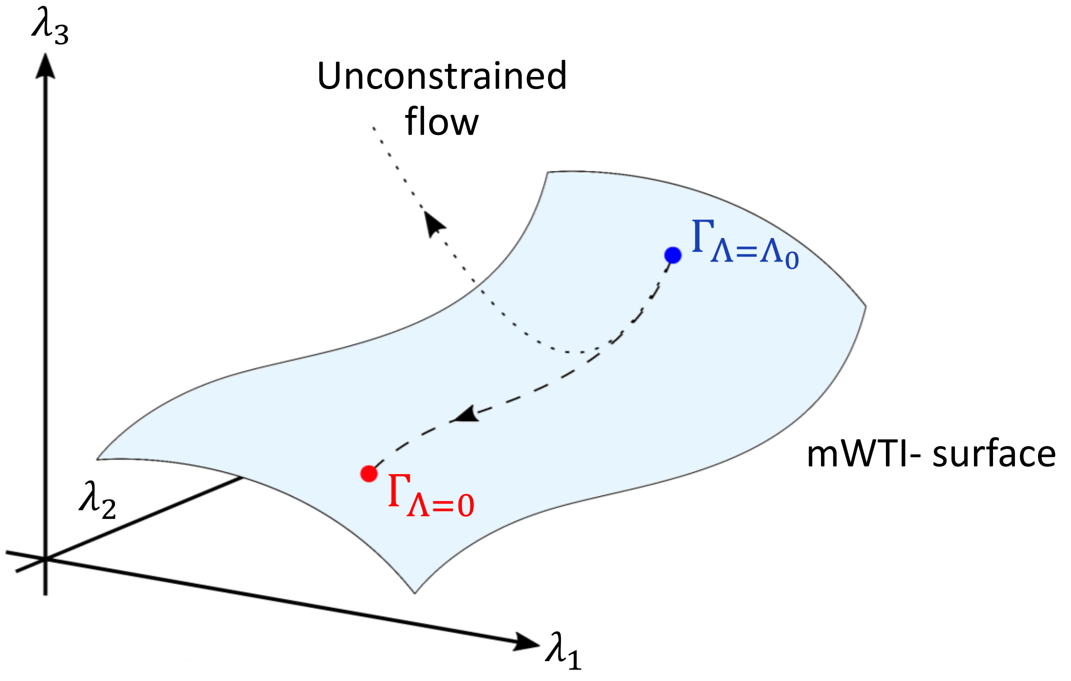
- Flow equations give the derivatives of $M_{\mathbf{A}}, M_\phi, g_{\mathbf{A}}, g_\phi, \mathcal{A}_\omega, \mathcal{A}_{\mathbf{k}}, \mathcal{B}_{\mathbf{A}\Omega}, \mathcal{B}_{\mathbf{Aq}}$ and $\mathcal{B}_{\phi\mathbf{q}}$.

- The mWTIs constrain $M_{\mathbf{A}}, M_\phi, g_{\mathbf{A}}$ and g_ϕ.

- Standard approach [16]: solve flow equations for independent variables $\mathcal{A}_\omega, \mathcal{A}_{\mathbf{k}}$, $\mathcal{B}_{\mathbf{A}\Omega}, \mathcal{B}_{\mathbf{Aq}}$ and $\mathcal{B}_{\phi\mathbf{q}}$, then fix $M_{\mathbf{A}}, M_\phi, g_{\mathbf{A}}$ and g_ϕ using the mWTIs.

[16] H. Gies, in "Renormalization Group and Effective Field Theory Approaches to Many-Body Systems", Chapter 6 (Springer-Verlag Berlin Heidelberg, 2012)

- This sums a larger class of diagrams [16,18] than using the flow-equations for $M_{\mathbf{A}}, M_\phi, g_{\mathbf{A}}$ and g_ϕ, and ensures we stay in the gauge-invariant subspace at all stages.



[16] Gies, in “Renormalization Group and Effective Field Theory Approaches to Many-Body Systems”, Chapter 6 (Springer-Verlag Berlin Heidelberg, 2012)
[18] Gies, Jaeckel & Wetterich, PRD **69**, 105008 (2004)

Outline

4. RG Fixed point

- Change variables to more convenient couplings [13,14]. Nine couplings:

$$\delta_{\mathbf{A}} = \frac{M_{\mathbf{A}}^2}{\mathcal{B}_{\mathbf{A}\Omega}\Lambda^2} \qquad \xi_{\mathbf{A}} = \sqrt{\frac{Ne^2}{8\pi\mathcal{B}_{\mathbf{A}\Omega}\Lambda}}$$

Also: $\mathcal{A}_\omega$

$$\delta_\phi = \frac{M_\phi^2\mathcal{A}_{\mathbf{k}}^2}{\mathcal{B}_{\phi\mathbf{q}}\mathcal{A}_\omega^2\Lambda^2} \qquad \xi_\phi = \sqrt{\frac{Ne^2}{8\pi\mathcal{B}_{\phi\mathbf{q}}\Lambda}}$$

$$g_{\mathbf{A}}'^2 = \frac{Ng_{\mathbf{A}}^2}{8\pi\mathcal{A}_{\mathbf{k}}^2\mathcal{B}_{\mathbf{A}\Omega}\Lambda} \qquad Y = \frac{\mathcal{A}_{\mathbf{k}}k_{UV}}{\mathcal{A}_\omega\Lambda}$$

$$g_\phi'^2 = \frac{Ng_\phi^2}{8\pi\mathcal{A}_\omega^2\mathcal{B}_{\phi\mathbf{q}}\Lambda} \qquad \zeta = \sqrt{\frac{\mathcal{B}_{\mathbf{A}\mathbf{q}}}{\mathcal{B}_{\mathbf{A}\Omega}}}\frac{\mathcal{A}_\omega}{\mathcal{A}_{\mathbf{k}}} \left(= \frac{c}{v}\right)$$

Recall: $N = \frac{k_F}{k_{UV}}$

- Y, $\xi_{\mathbf{A}}$ and ξ_ϕ are defined in order to remove non-universal dependence of the flow equations/mWTIs on k_{UV} and e (dimensionful in 2-D).

[13] Maier & Strack, PRB **93**, 165114 (2016) [14] Trott & Hooley, PRB **98**, 201113(R) (2018)

Recall:

$$\delta_{\mathbf{A}} = \frac{M_{\mathbf{A}}^2}{\mathcal{B}_{\mathbf{A}\Omega}\Lambda^2} \qquad g_{\mathbf{A}}'^2 = \frac{N g_{\mathbf{A}}^2}{8\pi \mathcal{A}_{\mathbf{k}}^2 \mathcal{B}_{\mathbf{A}\Omega}\Lambda}$$

$$\delta_{\phi} = \frac{M_{\phi}^2 \mathcal{A}_{\mathbf{k}}^2}{\mathcal{B}_{\phi\mathbf{q}}\mathcal{A}_{\omega}^2\Lambda^2} \qquad g_{\phi}'^2 = \frac{N g_{\phi}^2}{8\pi \mathcal{A}_{\omega}^2 \mathcal{B}_{\phi\mathbf{q}}\Lambda}$$

- In terms of these variables, the mWTIs are

$$\delta_{\mathbf{A}} = 2 g_{\mathbf{A}}' \xi_{\mathbf{A}} \qquad \delta_{\phi} = 8 g_{\phi}' Y \xi_{\phi}$$

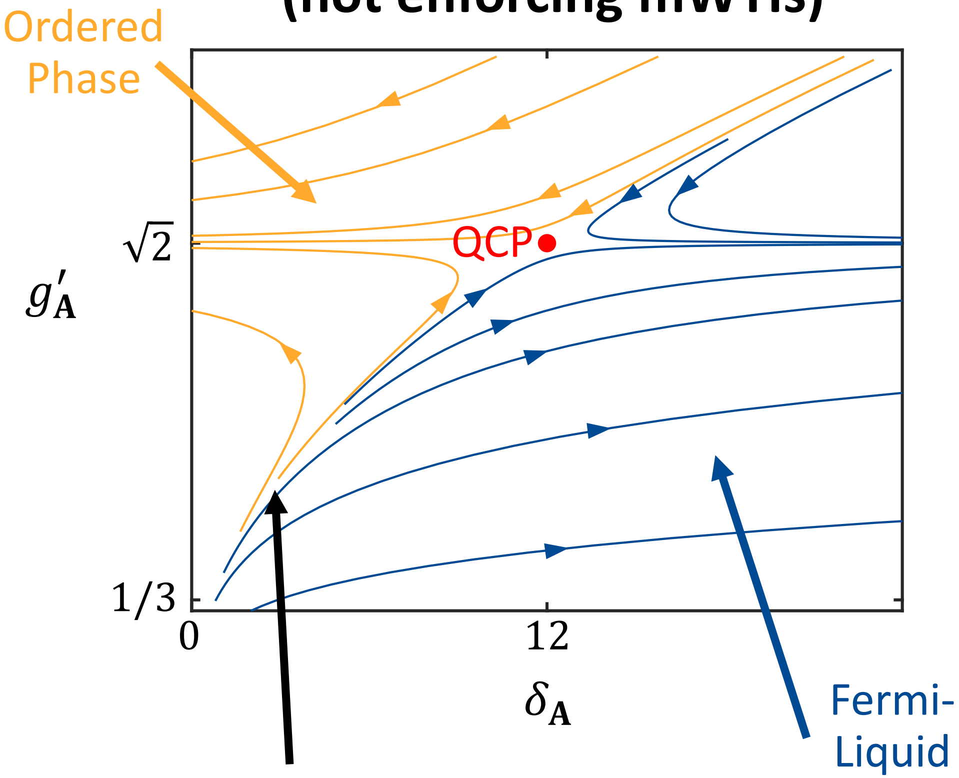
$$g_{\mathbf{A}}' = \xi_{\mathbf{A}} \left[1 + \frac{1}{N} \left({g_{\phi}'}^2 \tilde{f}_1 - g_{\mathbf{A}}'^2 \tilde{f}_2 \right) \right] \qquad g_{\phi}' = \xi_{\phi} \left[1 + \frac{1}{N} \left({g_{\phi}'}^2 \tilde{f}_3 - g_{\mathbf{A}}'^2 \tilde{f}_4 \right) \right]$$

Complicated functions

N.B. the g_ϕ' - identity originally contained extra $\propto 1/N$ terms (due to the frequency-cutoff) that drove g_ϕ' and δ_ϕ negative. We've dropped these in the expectation that they are unphysical, and would be cancelled in a more sophisticated treatment.

Unconstrained

(not enforcing mWTIs)

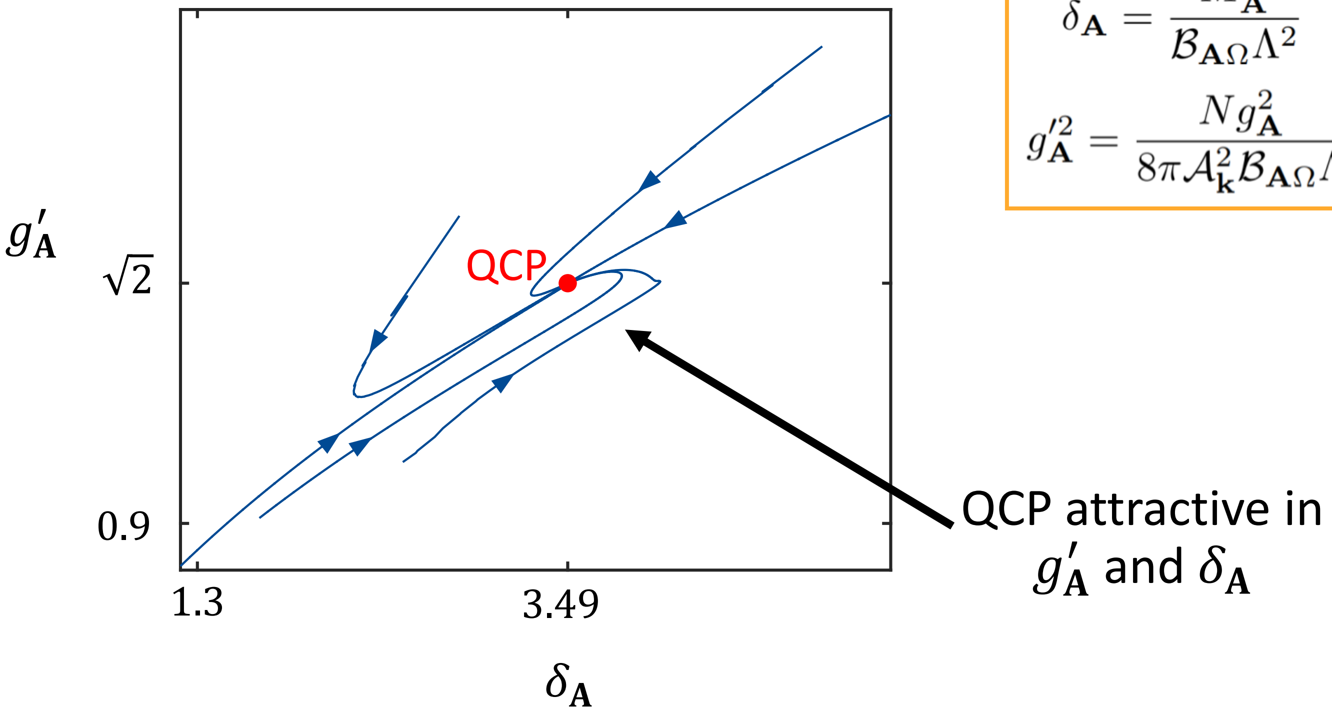


Need to tune $\delta_{\mathbf{A}}$ to reach criticality

Constrained

(enforcing mWTIs)

$$\delta_{\mathbf{A}} = \frac{M_{\mathbf{A}}^2}{\mathcal{B}_{\mathbf{A}\Omega}\Lambda^2}$$
$$g_{\mathbf{A}}'^2 = \frac{N g_{\mathbf{A}}^2}{8\pi \mathcal{A}_{\mathbf{k}}^2 \mathcal{B}_{\mathbf{A}\Omega}\Lambda}$$



- Can't break gauge symmetry (need higher-form symmetries to understand ordering) → mWTIs correctly force $\delta_{\mathbf{A}}$ to be irrelevant about the fixed point.

- Most of the fixed-point values for the couplings are not changed much by symmetry-constraints:

$$\eta_{\omega/\mathbf{k}} = -\Lambda\, \partial_\Lambda \log \mathcal{A}_{\omega/\mathbf{k}}$$
$$\zeta = c/v$$

Unconstrained

$$\eta_\omega = \frac{1}{2} - 0.262\,\frac{\log N}{N} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$\eta_{\mathbf{k}} = -0.024\,\frac{\log N}{N} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$\zeta^* = \frac{0.314}{N} + \mathcal{O}\left(\frac{\log N}{N^2}\right)$$

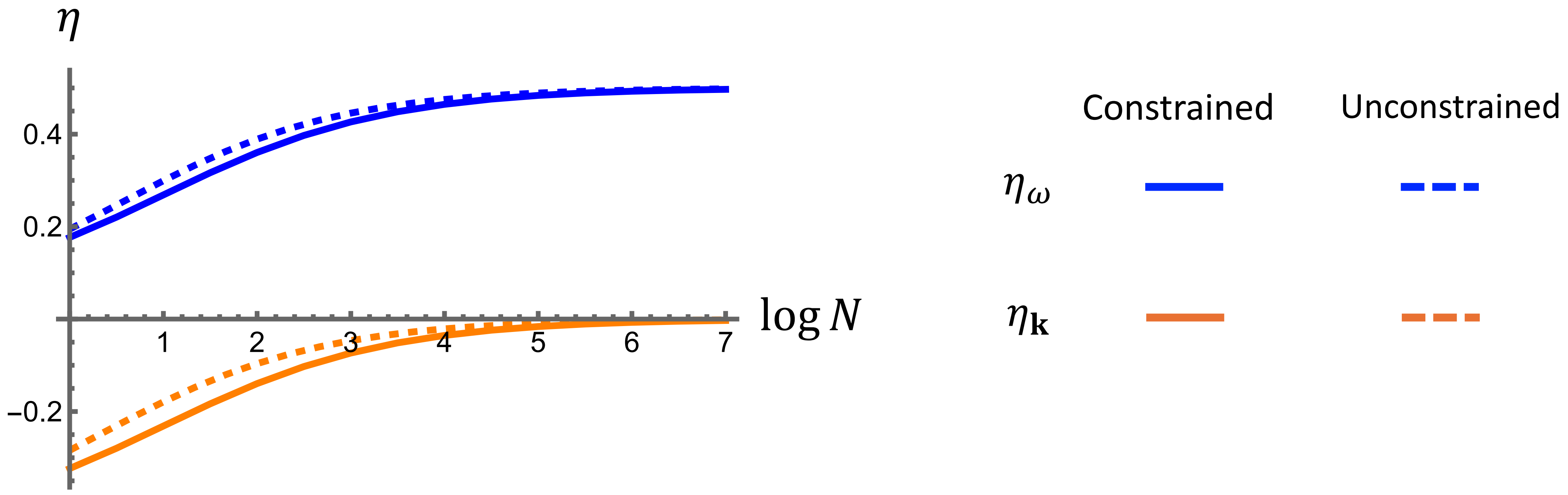
Constrained

$$\eta_\omega = \frac{1}{2} - 0.467\,\frac{\log N}{N} + \mathcal{O}\left(\frac{1}{N}\right)$$

$$\eta_{\mathbf{k}} = -0.467\,\frac{\log N}{N} + \mathcal{O}\left(\frac{1}{N}\right)$$

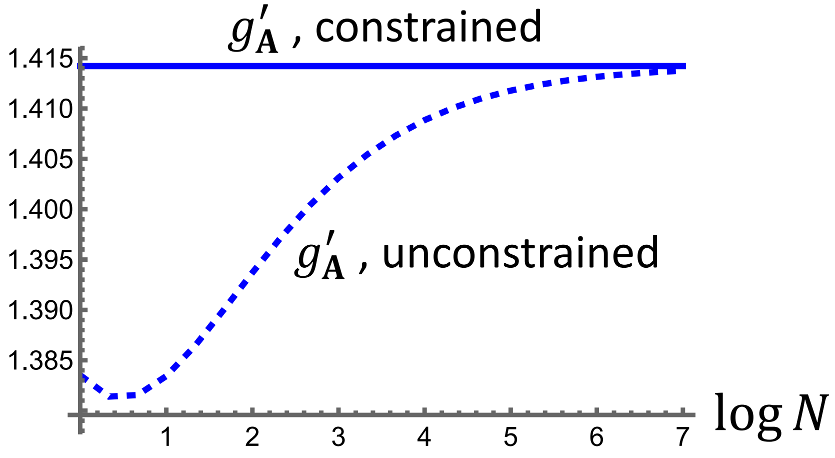
$$\zeta^* = \frac{0.857}{N} + \mathcal{O}\left(\frac{\log N}{N^2}\right)$$

Note for experts: neglected the feedback of fermion anomalous dimensions on the RHS of flow equations – corrections should be small.


η
0.4
0.2
−0.2
1
2
3
4
5
6
7
$\log N$
Constrained
Unconstrained
η_ω
$\eta_{\mathbf{k}}$

- However, there are some more noticeable differences:

$g'_{\mathbf{A}} = \sqrt{2} - 0.337\frac{\log N}{N} + \mathcal{O}\left(\frac{1}{N}\right)$ for the unconstrained case, but is identically $\sqrt{2}$ for the constrained.

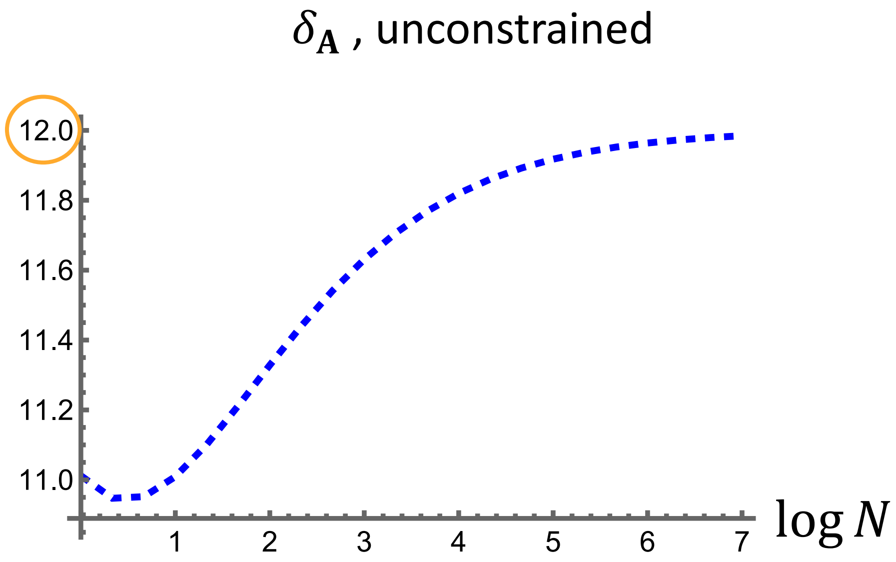


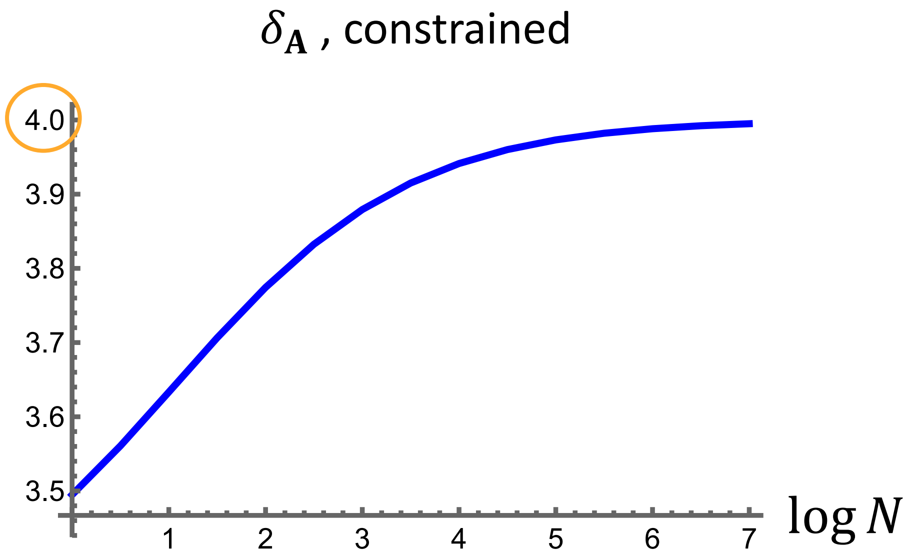
- Also, $\xi_{\mathbf{A}} \to \infty$ in the unconstrained case, but $\to \sqrt{2} + \mathcal{O}\left(\frac{\log N}{N}\right)$ in the constrained.

$$\Rightarrow \mathcal{B}_{\mathbf{A}\Omega} \sim \frac{Ne^2}{16\pi\Lambda}$$

$$\boxed{\xi_{\mathbf{A}} = \sqrt{\frac{Ne^2}{8\pi\mathcal{B}_{\mathbf{A}\Omega}\Lambda}}}$$

- Finally: $\delta_{\mathbf{A}} \xrightarrow{N\to\infty} 12$ and 4 for unconstrained/constrained, respectively.

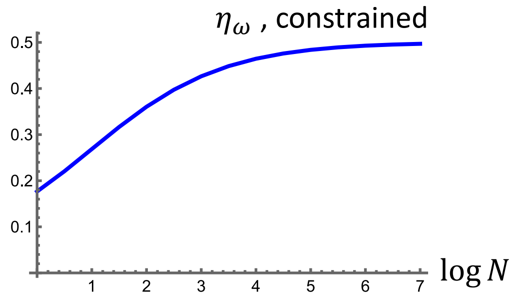




- $\delta_{\mathbf{A}}^{-1} = 1/12$ almost under perturbative control?

- How does this compare to past work?

	$z_{\mathbf{A}}$	$\Sigma(\omega, k_F)$
This work	2	$\sim \omega^{1-\eta_\omega}$
Hertz-Millis-type [19,20]	3	$\sim \omega^{2/3}$
H-M-type, >3-loop [9]	> 3	$\sim \omega^{>2/3}$



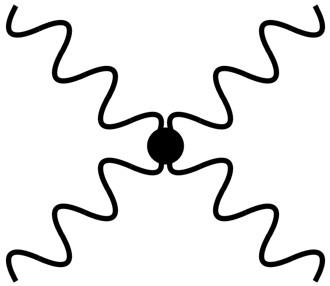
[19] Polchinski, Nucl. Phys. B **422**, 617 (1994) [20] Kim, Furusaki, Wen & Lee, **PRB** 50, 17917 (1994)
[9] Holder & Metzner, PRB **92**, 041112(R) (2015)

Outline

Possible implications for predictive modelling of NFLs

- Apparent that gauge-symmetry constraints don't change the properties of our model a lot. However, there are some changes, so this acts as a proof-of-principle for the method.
- Can we envisage a model in which there is a bigger difference?
  - The fact that $g'_{\mathbf{A}} \approx \sqrt{2}$ in both cases seems to be a bit of an "accident", arising due to $\partial_\Lambda \mathcal{B}_{\mathbf{Aq}} \to 0$ at criticality.

    A more sophisticated treatment, with a 4-boson vertex, might fix this.



  - More speculative: generalize the gauge group to $\mathrm{SU}(N)$? → "richer" mWTIs.

- Applying our type of analysis (fRG + symmetry-constraints) may also work better for theories with emergent (global) symmetries, where the symmetry group is much larger/non-compact (e.g. $\mathrm{LU}(1)$).

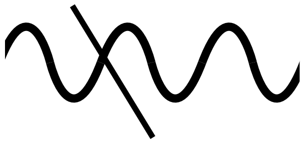
  Indeed, there is an exact solution of the Tomonaga-Luttinger model by fRG that uses the emergent $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry of the Luttinger liquid! [21] (highly recommend!)

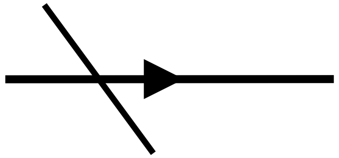
[21] Schütz, Bartosch, Kopietz, **PRB** 72, 035107 (2005)

Summary

- Much modelling of non-Fermi liquids suffers from uncontrolled approximations, which limits predictive power.
- Partial solution: utilize exact constraints, such as those provided by (gauge) symmetries, to constrain modelling.
- Simplest example: $\mathrm{U}(1)$ gauge field interacting with circular Fermi surface.
- Challenge: reproduce Landau damping while preserving the symmetry.
- The functional renormalization group with a soft fermionic frequency cutoff nonperturbatively produces flow equations and lets Landau damping develop smoothly.
- Gauge symmetry is enforced by modified Ward-Takahashi identities.
- The model has an NFL fixed point, with $z_{\mathbf{A}} = 2$ and $\eta_\omega \approx 1/2$. The anomalous dimensions and couplings are somewhat affected by the symmetry-constraints. Gauge symmetry also makes the boson mass irrelevant $\rightarrow$ conventional ordering forbidden.

- Single-scale propagators for flow equations:

 $= \partial_\Lambda^R G_b(\Omega, \mathbf{q})$

 $= \partial_\Lambda^R G_f(\omega, \mathbf{k})$

$$\left.\begin{array}{l} = \partial_\Lambda^R G_b(\Omega, \mathbf{q}) \\ = \partial_\Lambda^R G_f(\omega, \mathbf{k}) \end{array}\right\} \partial_\Lambda^R = \sum_i \partial_\Lambda R_i \frac{\partial}{\partial R_i} \quad , \qquad R_i = \left(G_{0,i}^\Lambda\right)^{-1} - G_{0,i}^{-1}$$

- “Special” propagator for mWTIs:

 $= \left(R_f(k+q) - R_f(k)\right) G_f(k) G_f(k+q)$ (with $k = (\omega, \mathbf{k})$, etc.)

(the boson “special” propagator plays no role for an Abelian gauge group)

- Why don't the bosonic "special" propagators appear in the mWTIs? Difficult to explain in full, but roughly:

Regulators appear in the effective action through $\int \bar{\psi}\, R_f\, \psi$ and $\int A_\mu\ R_b^{\mu\nu}\ A_\nu$.
Under an infinitesimal gauge transformation, these change as

$$\delta_\alpha\left(\int \bar{\psi}\, R_f\, \psi\right) = i\int \bar{\psi}[\alpha, R_f]\,\psi$$

$$\delta_\alpha\left(\int A_\mu\ R_b^{\mu\nu}\ A_\nu\right) = \frac{1}{e}\int \left(\partial_\mu\alpha\, R_b^{\mu\nu} A_\nu + A_\mu\, R_b^{\mu\nu}\ \partial_\nu\alpha\right)$$

The fermion-term is quadratic in ψ. Changing $\psi \to \delta/\delta\bar{\eta}$, etc. generates a trace
→ one-loop diagram.
The boson term is linear → no contribution.